Using Mean-Variance Optimization in the Real World: Black-Litterman vs. Resampling

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Making Mean-Variance Optimization Usable

- Mean-Variance Optimization (MVO) has been little used in practice.
- Both Black-Litterman and Resampling, when combined with MVO, create more diversified portfolios.
- Only Black-Litterman creates intuitive portfolios that are usable in the real world.
- Portfolios on the resampled frontier include active risk caused by the forecasts and averaging process.

MVO and the Asset Allocation Process

- Mean-Variance Optimization leads to unintuitive, undiversified portfolios.
- Until recently, MVO has mostly been used as window dressing.

MVO, though a powerful algorithm, has not found its place in practical asset allocation.

The Power of MVO

- Mean-Variance Optimization was developed by Nobel Laureate Harry Markowitz in 1952.
- Markowitz discovered that an investor can reduce the volatility of a portfolio and increase its return at the same time.
- Diversification: The risk of a portfolio can be decreased by combining assets whose returns move in different directions under certain market conditions.

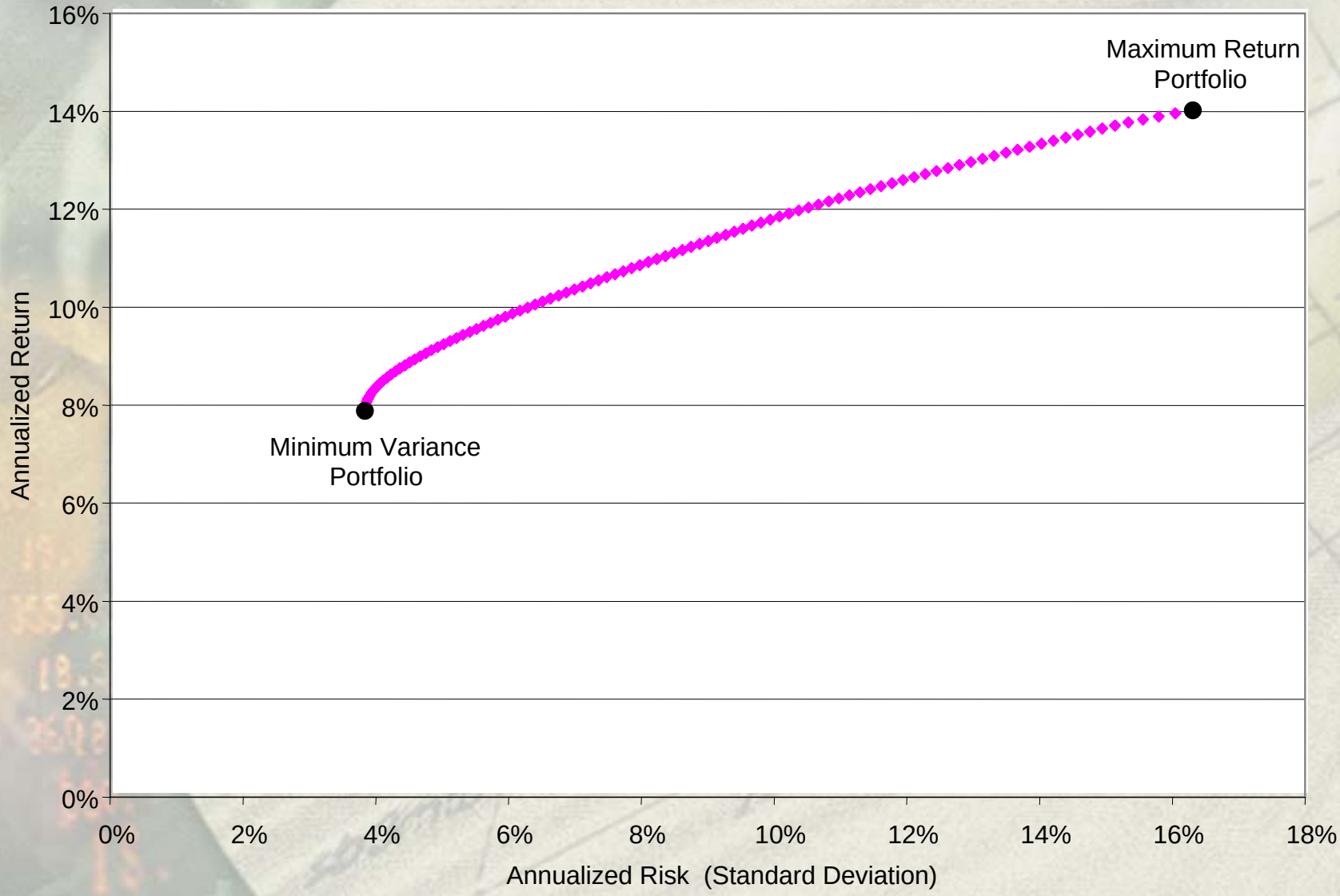
MVO in Two Stages

1. Calculate the forecasts.
   - Calculate forecasts for returns, standard deviations and correlations for the set of assets in which you can invest.
   - This is often done using historical data.
2. Calculate the Efficient Frontier.
   - The efficient frontier is the set of portfolios that minimizes risk at the possible levels of return.
   - A portfolio can be selected from the frontier based on risk, utility maximization, maximum Sharpe Ratio, etc.

The Mechanics

1. Create or calculate Forecasts for Return, Risk and Correlations for a set of assets. These parameters describe a multivariate return distribution.
2. Calculate the Efficient Frontier.
   - Assume that all portfolios have positive weights (no short-selling) and add to 100.
   - Calculate the minimum variance portfolios and maximum return portfolio using the forecasts.
   - Calculate the portfolio that minimizes risk for each of 98 portfolios between the minimum variance and maximum return portfolios. This set of 100 portfolios is the efficient frontier.

The Efficient Frontier



Limitations of MVO

- Returns are very difficult to forecast.
  - MVO requires forecasts on ALL assets.
  - Historical returns are very poor forecasts.
- Input Sensitivity--MVO is highly sensitive to the return forecasts.
  - Small changes in return assumptions often lead to large changes in the optimal allocations.

Estimation Error is built into forecasting and magnified by MVO

Estimation Error Leads to Unusable Portfolios

- Portfolios are very concentrated (no diversification).
- Portfolios are unintuitive.

Both of these issues must be solved to make MVO a practical real-world tool.

Two Approaches to Creating Diversified Portfolios with MVO

- Black-Litterman
  - Technique developed by Fischer Black and Robert Litterman of Goldman Sachs to create better return estimates.
- Resampling
  - Technique developed by Richard Michaud to average over the statistical equivalence region and create a new efficient frontier.

An Experiment to Compare the Two Techniques

- Select a set of assets.
- Calculate an efficient frontier using Historical Inputs, Resampling and Black-Litterman Inputs.
- Compare the resulting portfolios.

The Assets

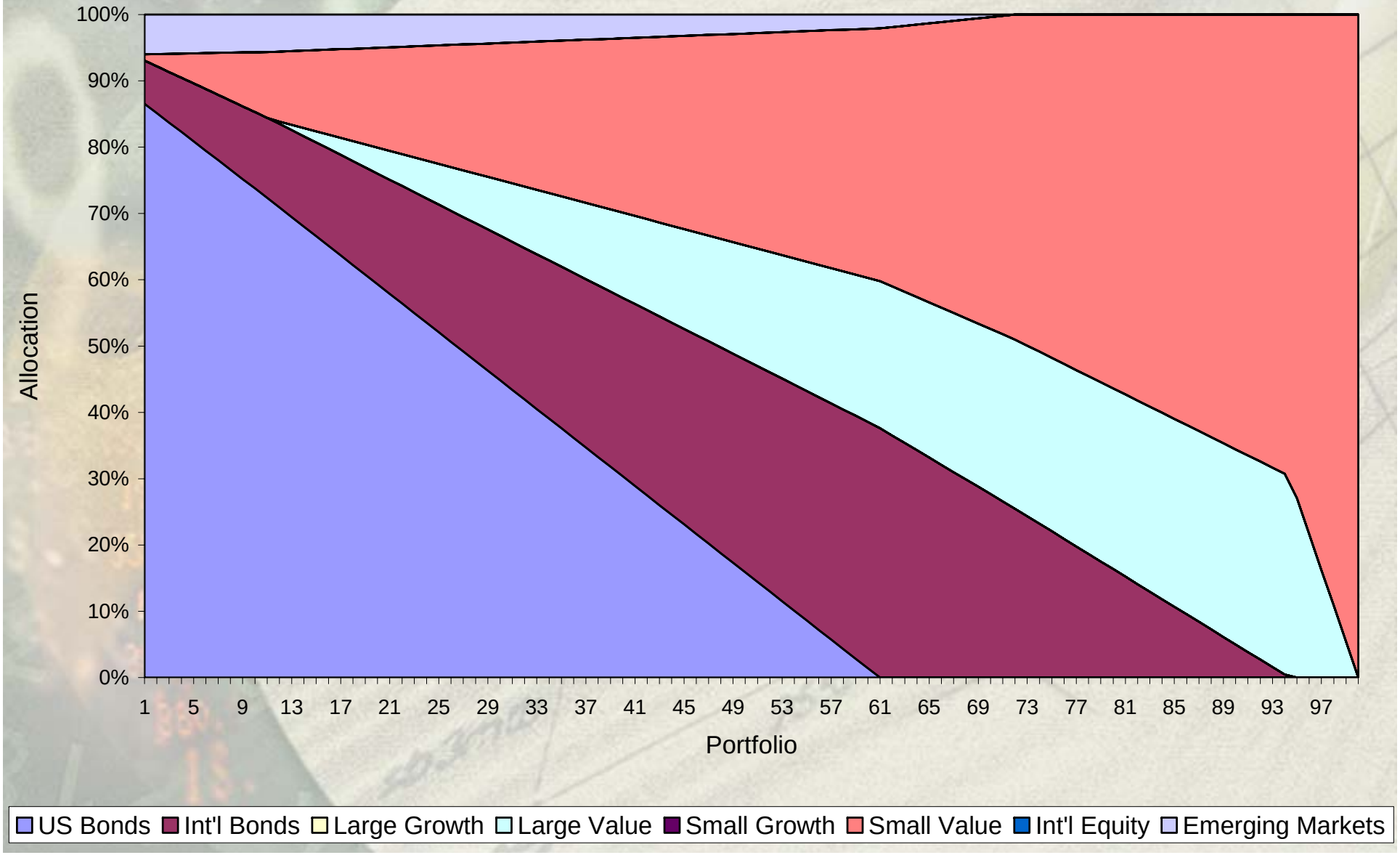
	Return	Std. Dev.
US Bonds	7.4%	4.2%
Int'l Bonds	8.4%	9.4%
Large Growth	11.8%	18.3%
Large Value	12.8%	14.2%
Small Growth	10.4%	24.0%
Small Value	14.0%	16.3%
Int'l Equity	8.4%	16.7%
Emerging Markets	12.5%	22.7%

Historical Data January 1987-July 2005

Are the Portfolios Diversified?

- First, let’s look at the diversification of the portfolios resulting from the three techniques.

Using Historical Forecasts in MVO Leads to Highly Concentrated Portfolios



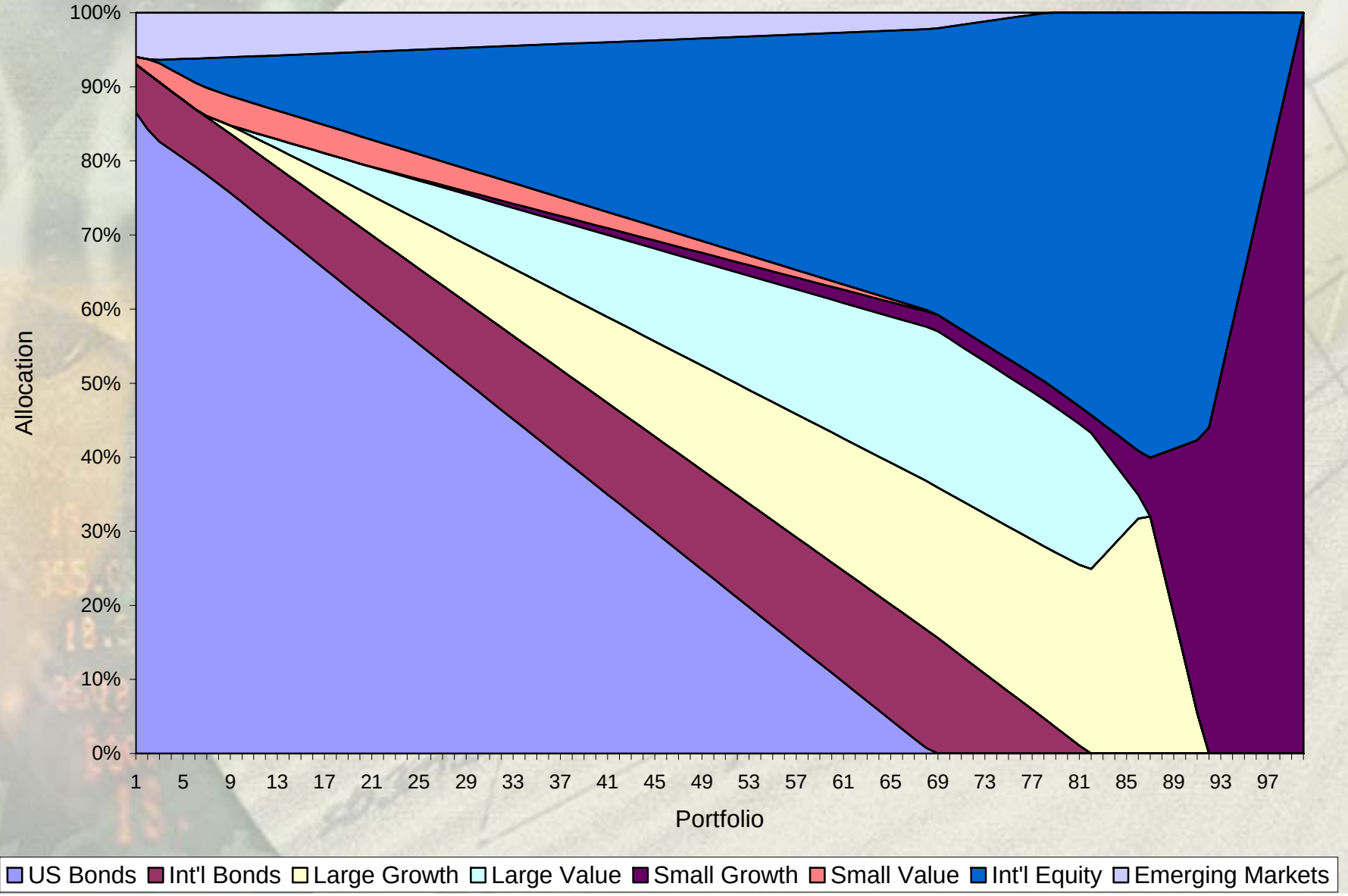
Black-Litterman Implied Returns

- Black-Litterman Implied Returns are consistent with MPT and CAPM.
- Black-Litterman Implied Returns are the returns that put the market in equilibrium.
- Black-Litterman Implied Returns are calculated using Reverse Optimization. The inputs are the market capitalizations and covariance matrix of the assets, and the risk premium for the set of assets.

Black-Litterman Returns as Forecasts

- Black-Litterman Implied Returns make excellent forecasts for use with MVO. The result is diversified, intuitive portfolios.

Black-Litterman Implied Returns Lead to Diversified Portfolios



Resampling

Stage 1 of MVO

1. Estimate returns, standard deviations and correlations for a set of assets. Michaud does this using historical data.
2. Run a Monte Carlo simulation, creating a new data set. Calculate the return, standard deviation and correlations of the new data set.

Stage 2 of MVO

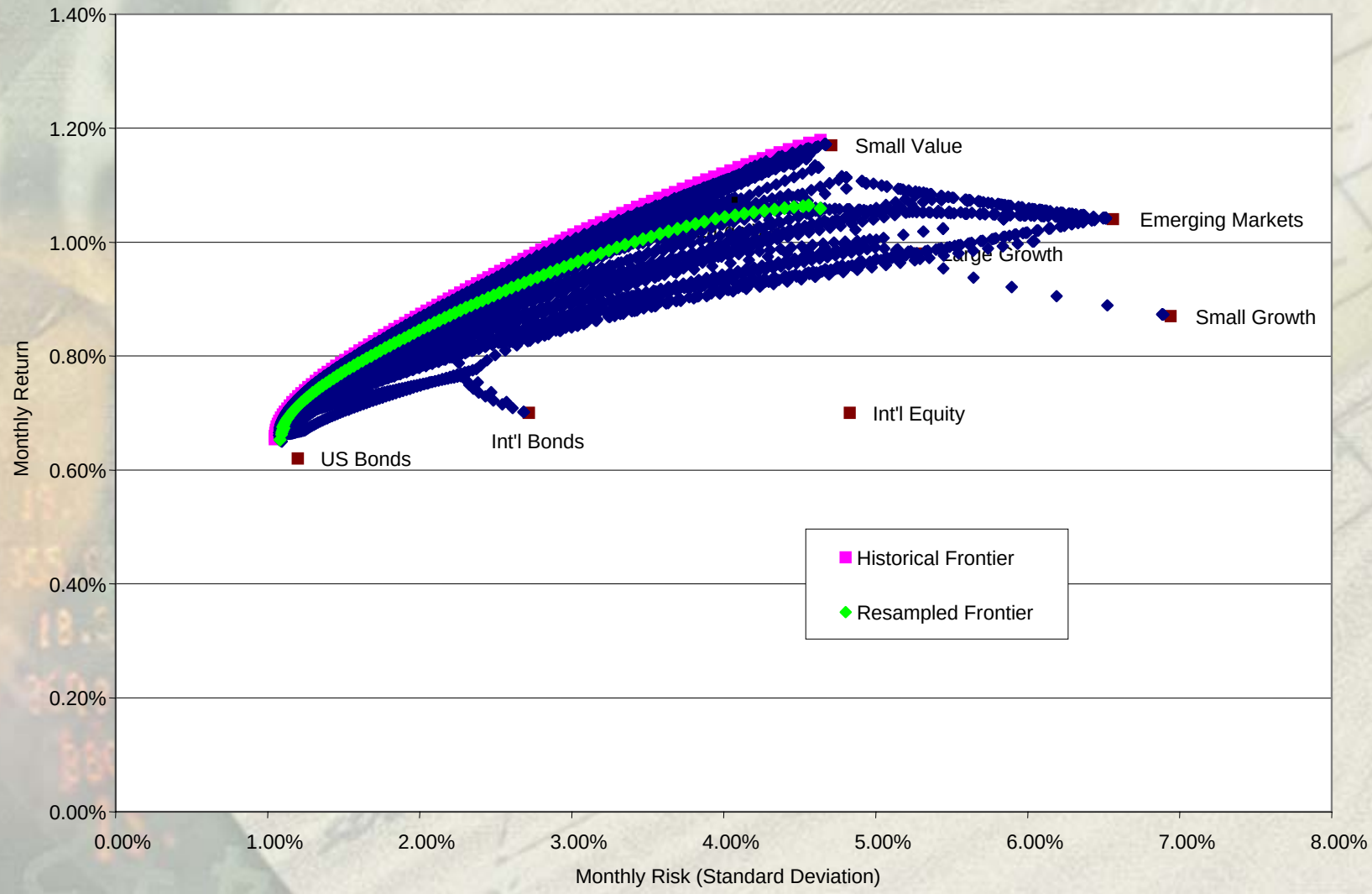
3. Create an efficient frontier using the new inputs.
4. Repeat steps 2 and 3 500 times.

Add'l Step

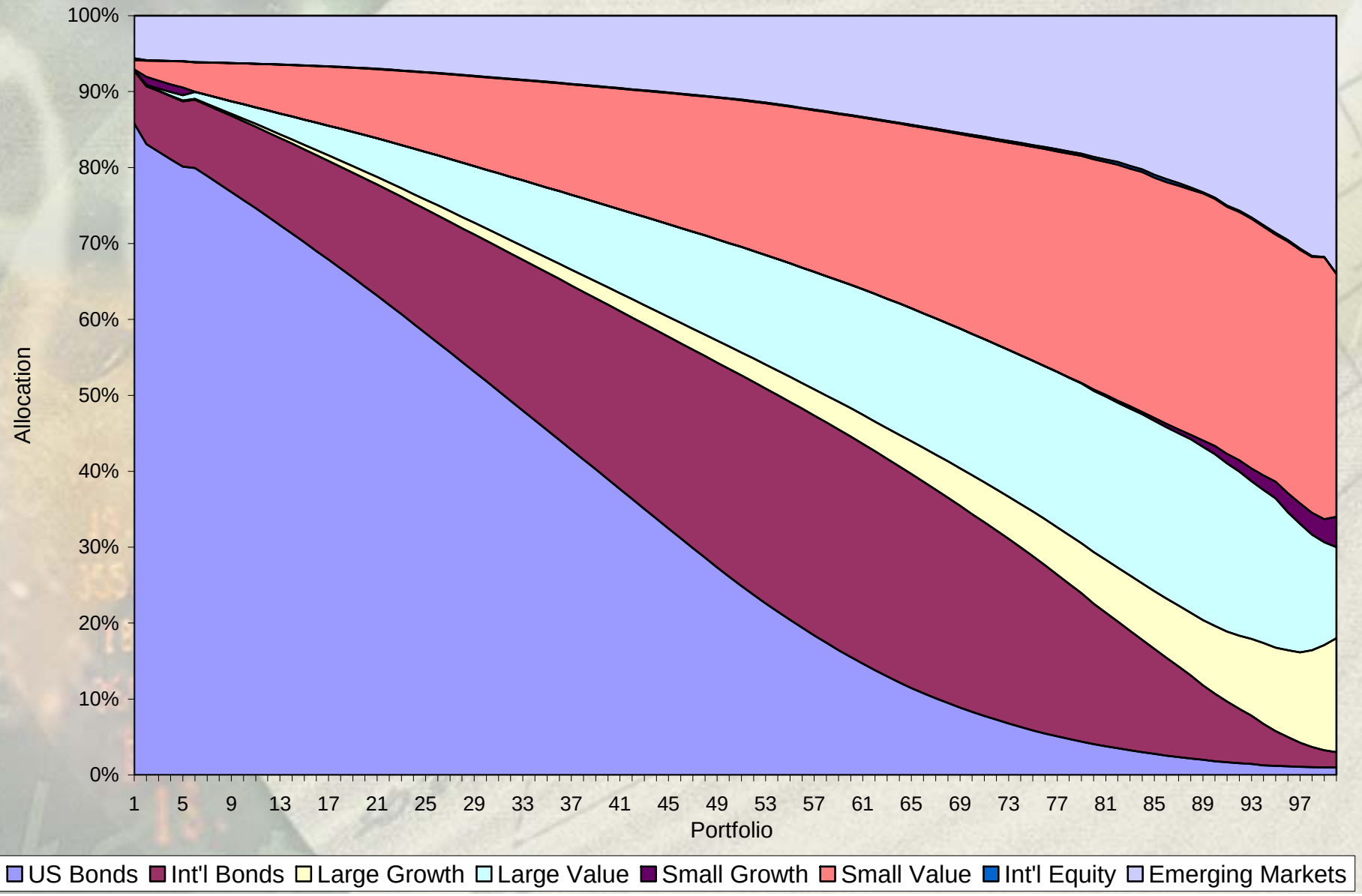
5. Calculate the average allocations to the assets for a set of predetermined return intervals. This is the new efficient frontier.

This procedure has U.S. Patent #6,003,018 by Michaud et al., December 12, 1999

The Resampled Frontier

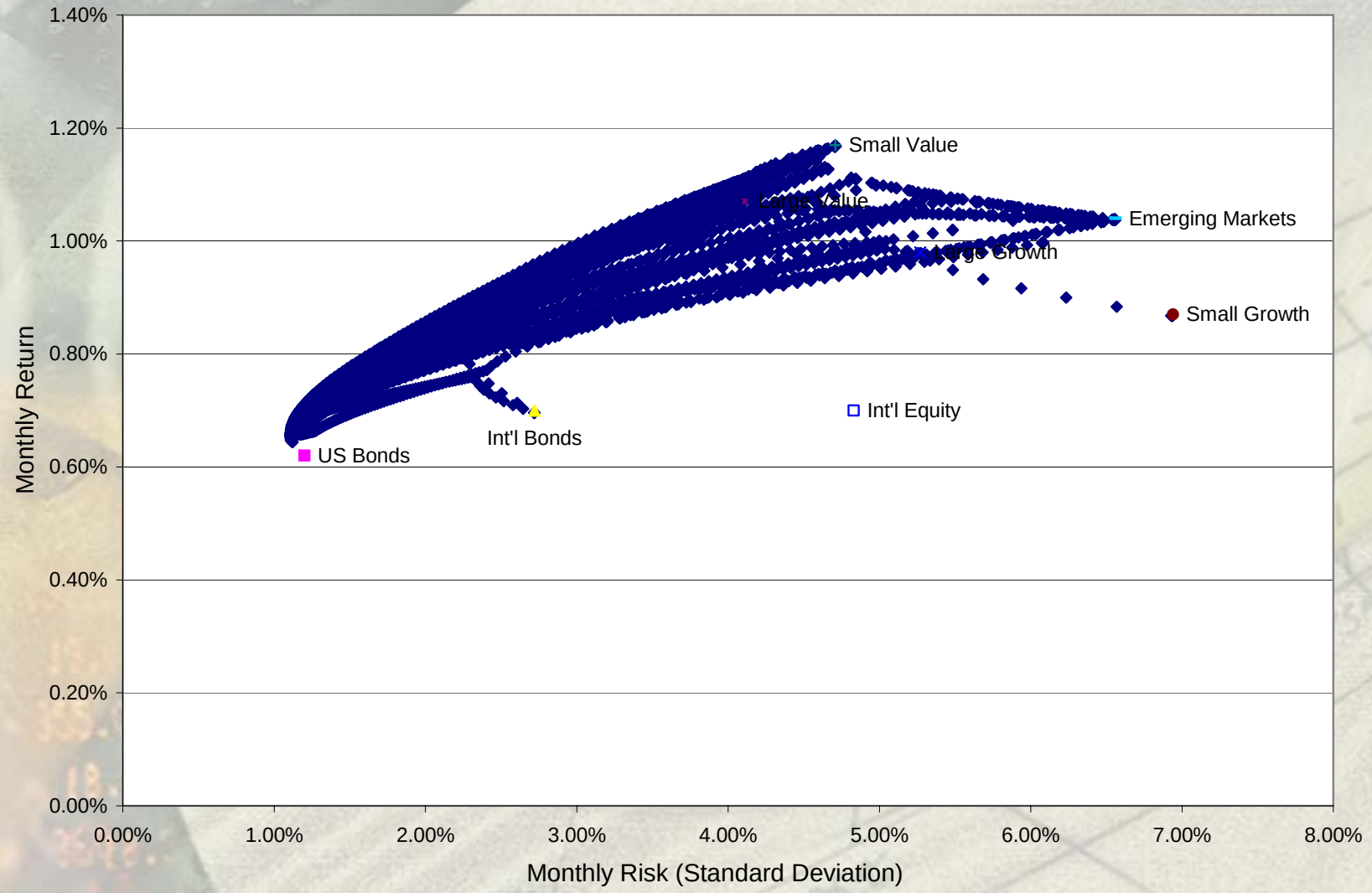


Resampling also Leads to Diversified Portfolios



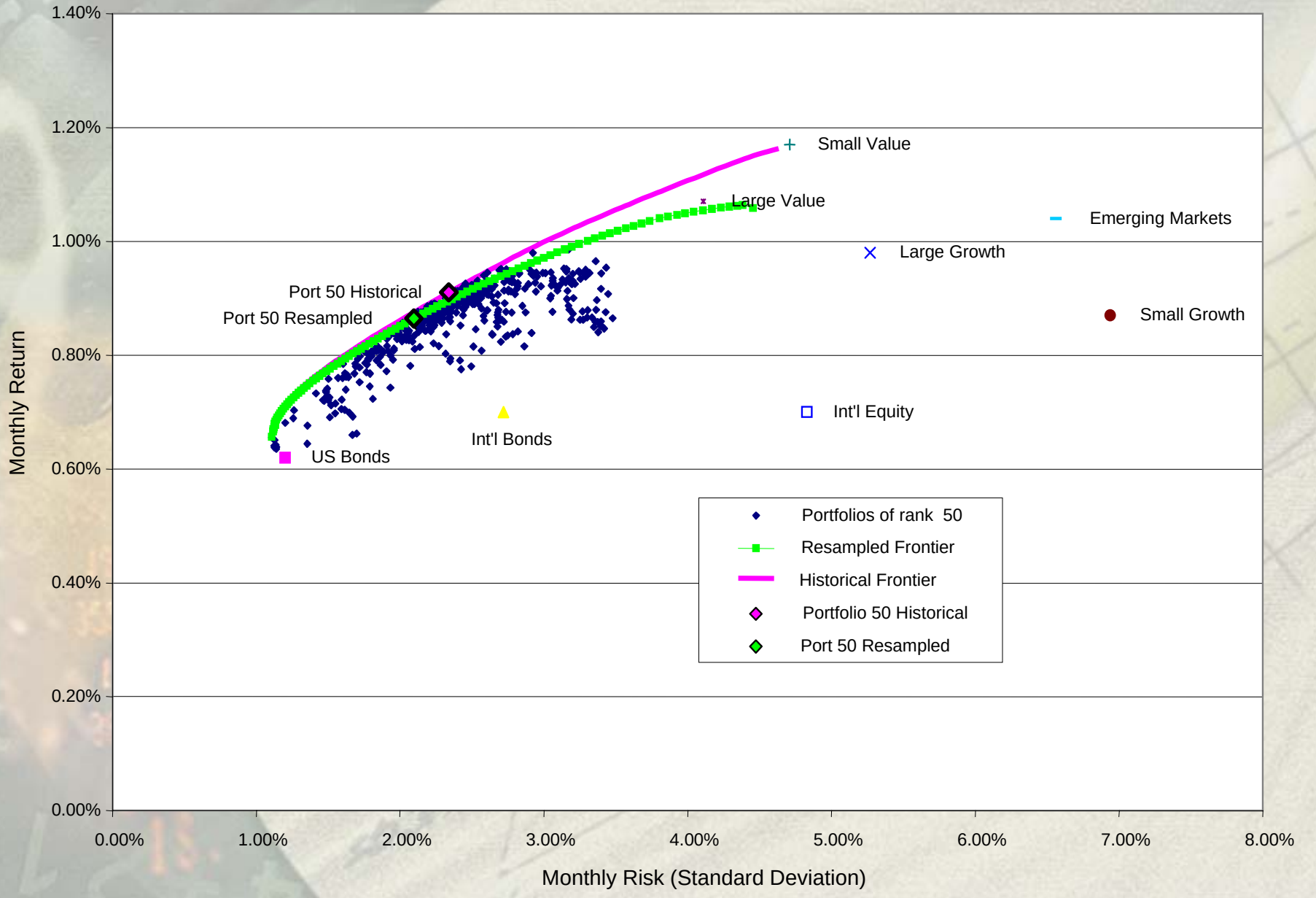
A Closer Look at the Resampled Frontier

Where is the Frontier?



Need to select one set of portfolios, but there is no theoretical motivation for Michaud's averaging

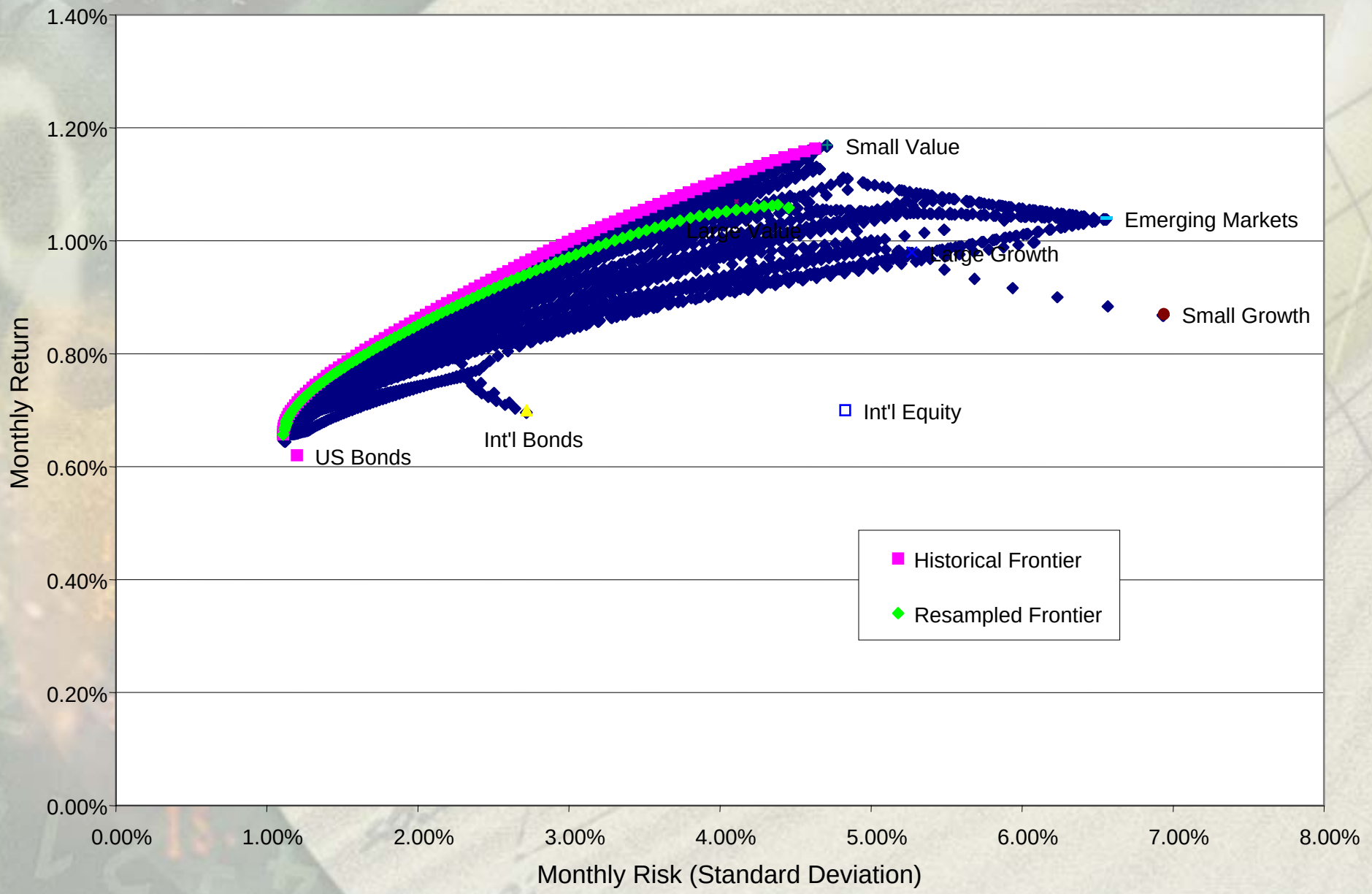
Portfolio #50



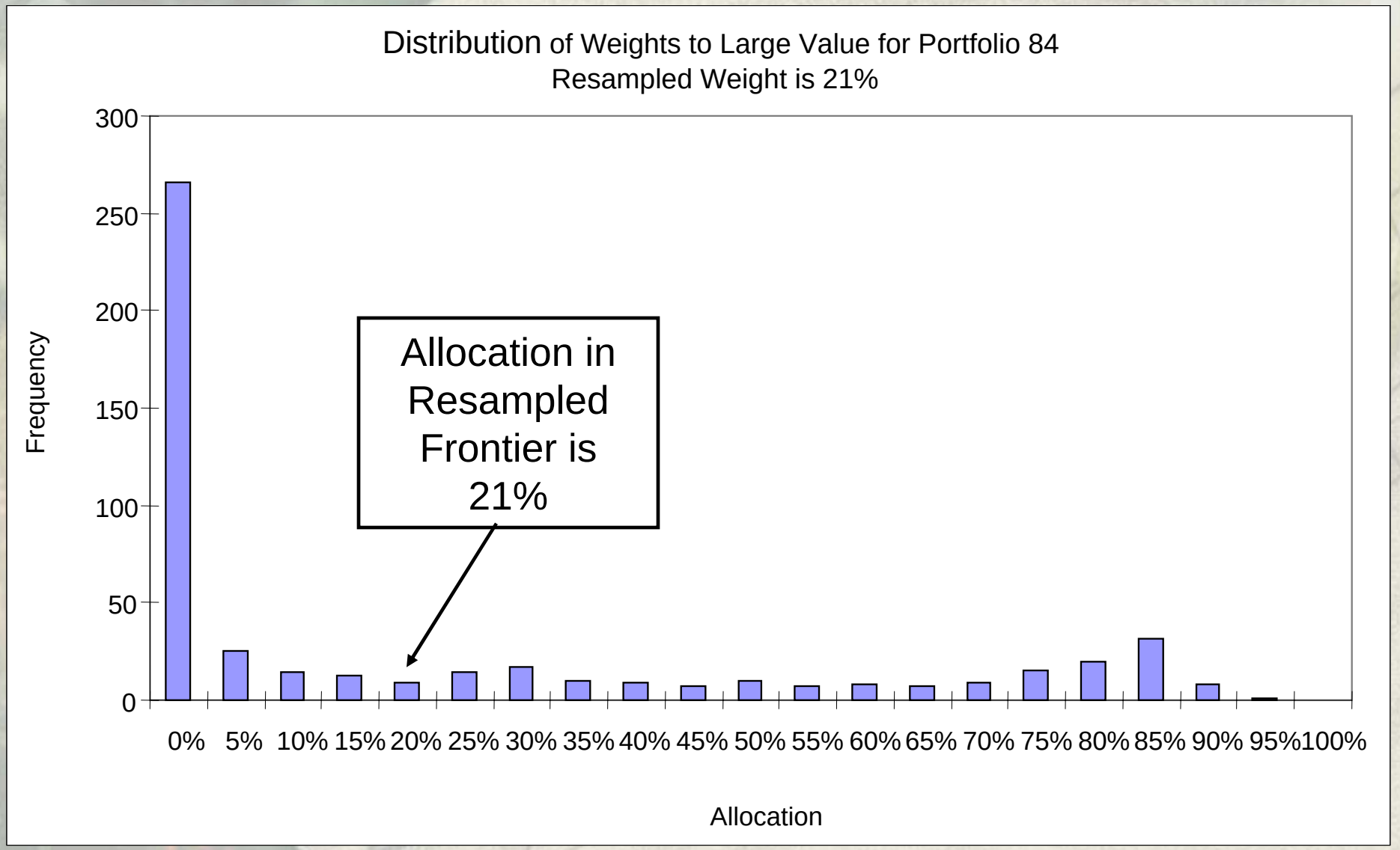
Consequences of Averaging to Create the Resampled Frontier

- Frontier is Suboptimal.
- Outliers tilt the allocations.
- Very small allocations to assets throughout frontier.
- It is possible to get an upward sloping frontier.

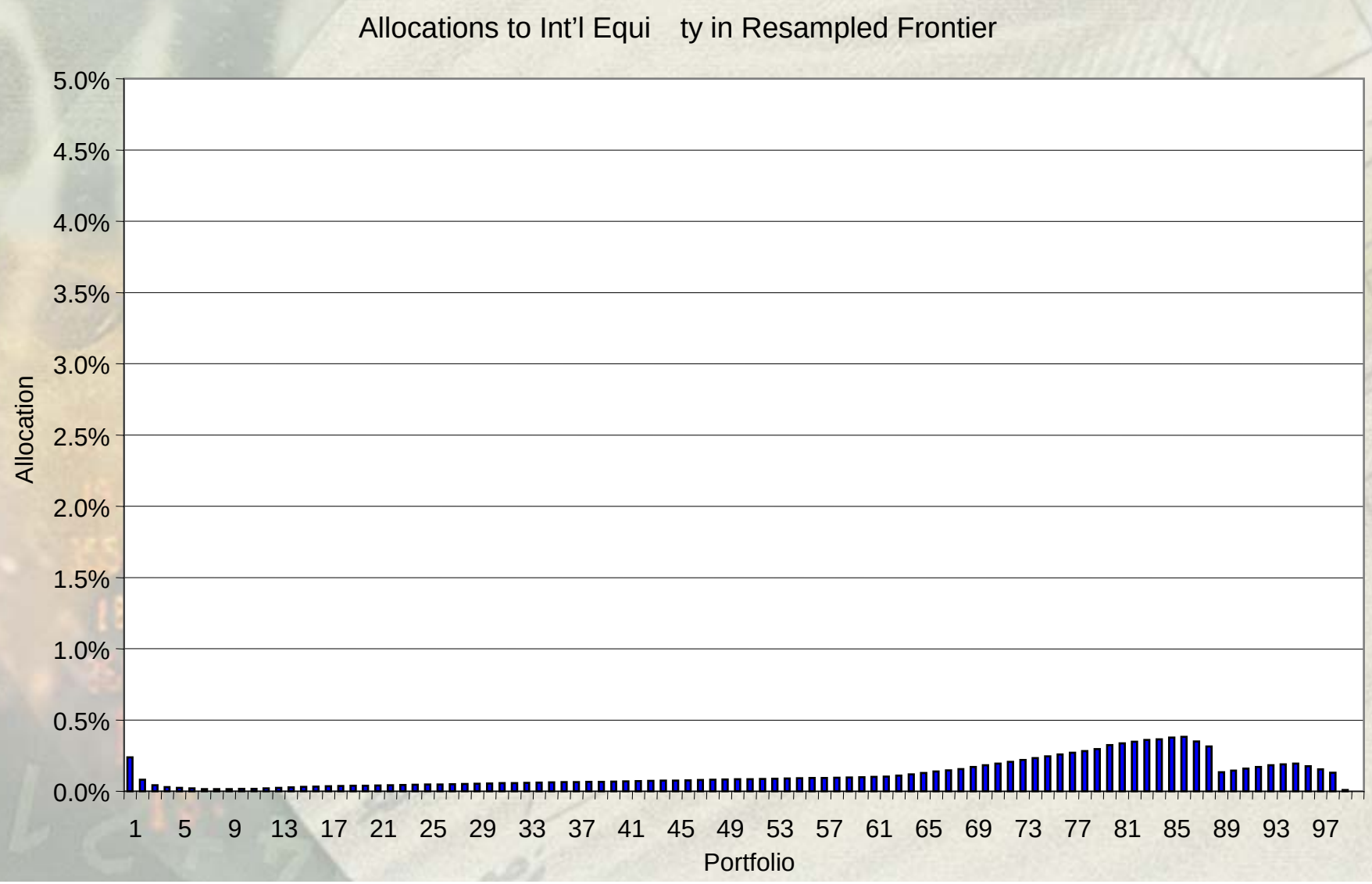
The Resampled Frontier Is Suboptimal



Frequencies and Averaged Weights



Allocations to Every Asset in Every Portfolio



Are the Portfolios Intuitive?

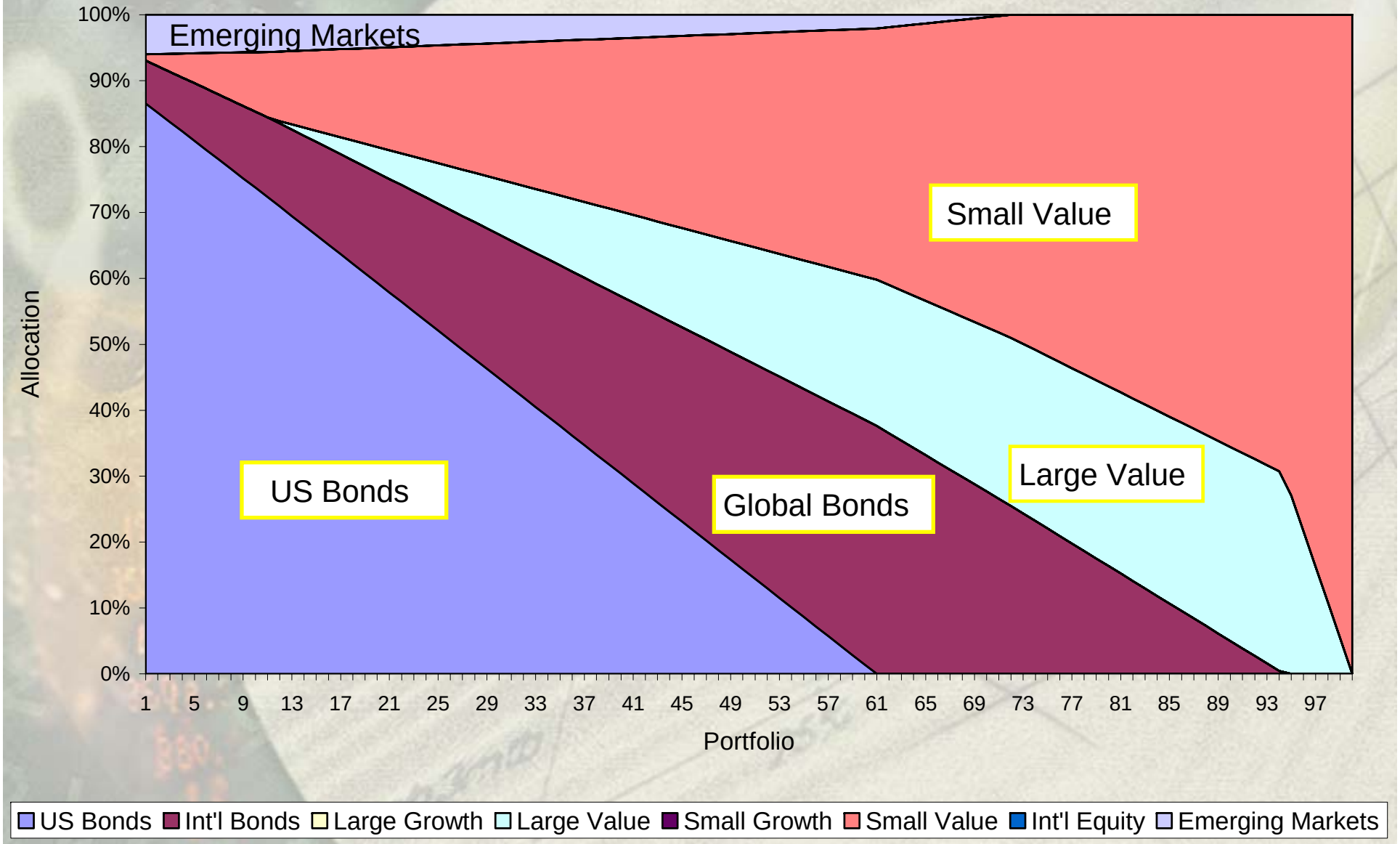
- Next, let's look at the allocations of the portfolios. Specifically, consider two questions:
  - Do the allocations make sense for real-world investment?
  - What kind of active risk would I be taking relative to a neutral asset allocation?

Historical Data

	Return	Risk	Sharpe Ratio
US Bonds	7.44%	4.16%	0.967
Int'l Bonds	8.40%	9.42%	0.529
Large Growth	11.76%	18.26%	0.457
Large Value	12.84%	14.24%	0.662
Small Growth	10.44%	24.04%	0.292
Small Value	14.04%	16.32%	0.651
Int'l Equity	8.40%	16.73%	0.298
Emerging Markets	12.48%	22.72%	0.399

January 1987-July 2005

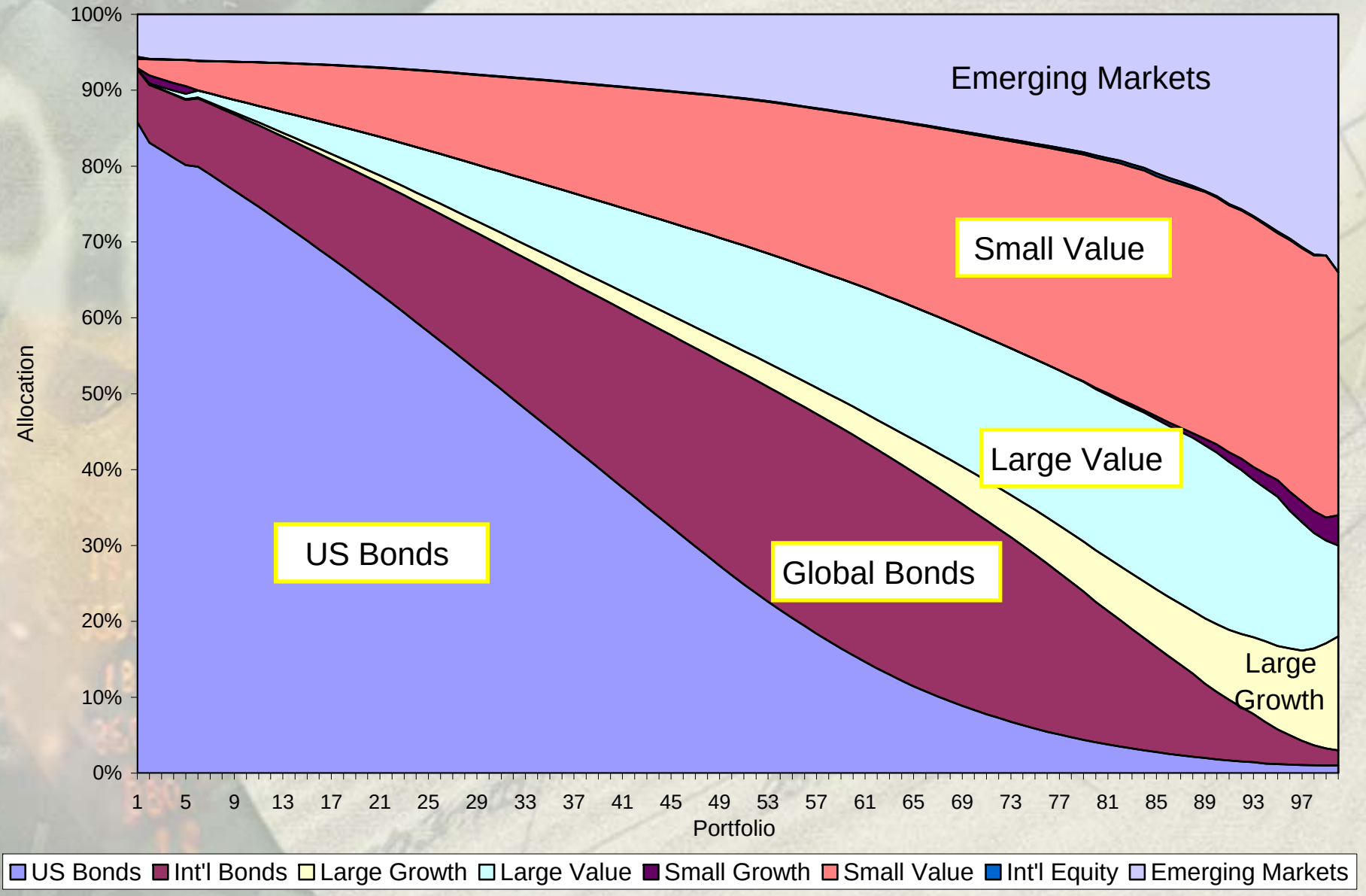
Historical Portfolios



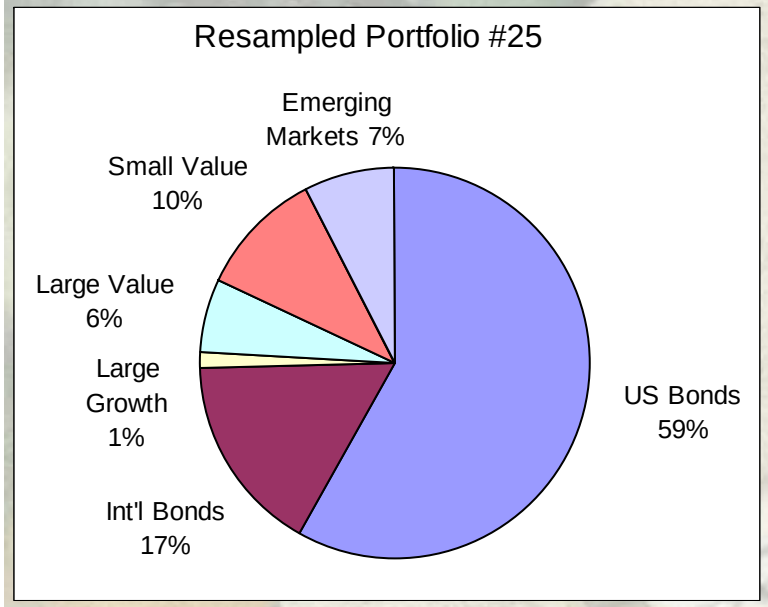
Forecasts and the Resampled Frontier

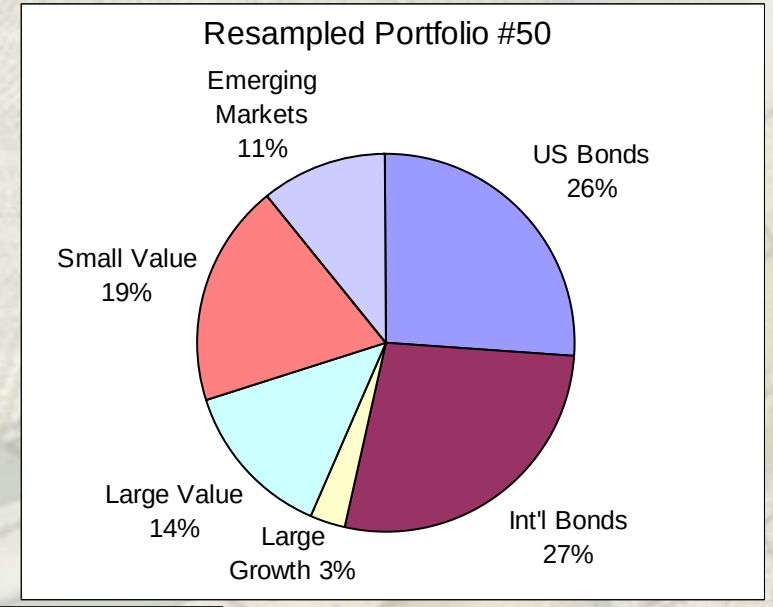
- The Portfolios from the Resampled Frontier are heavily influenced by the original forecasts.
- Remember, making forecasts is hard.

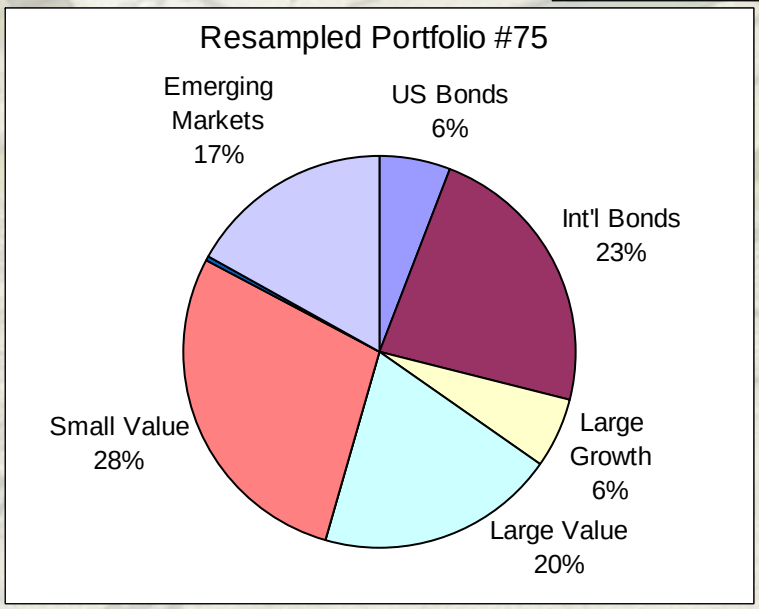
Resampled Portfolios



Do the Resampled Portfolios Make Sense?



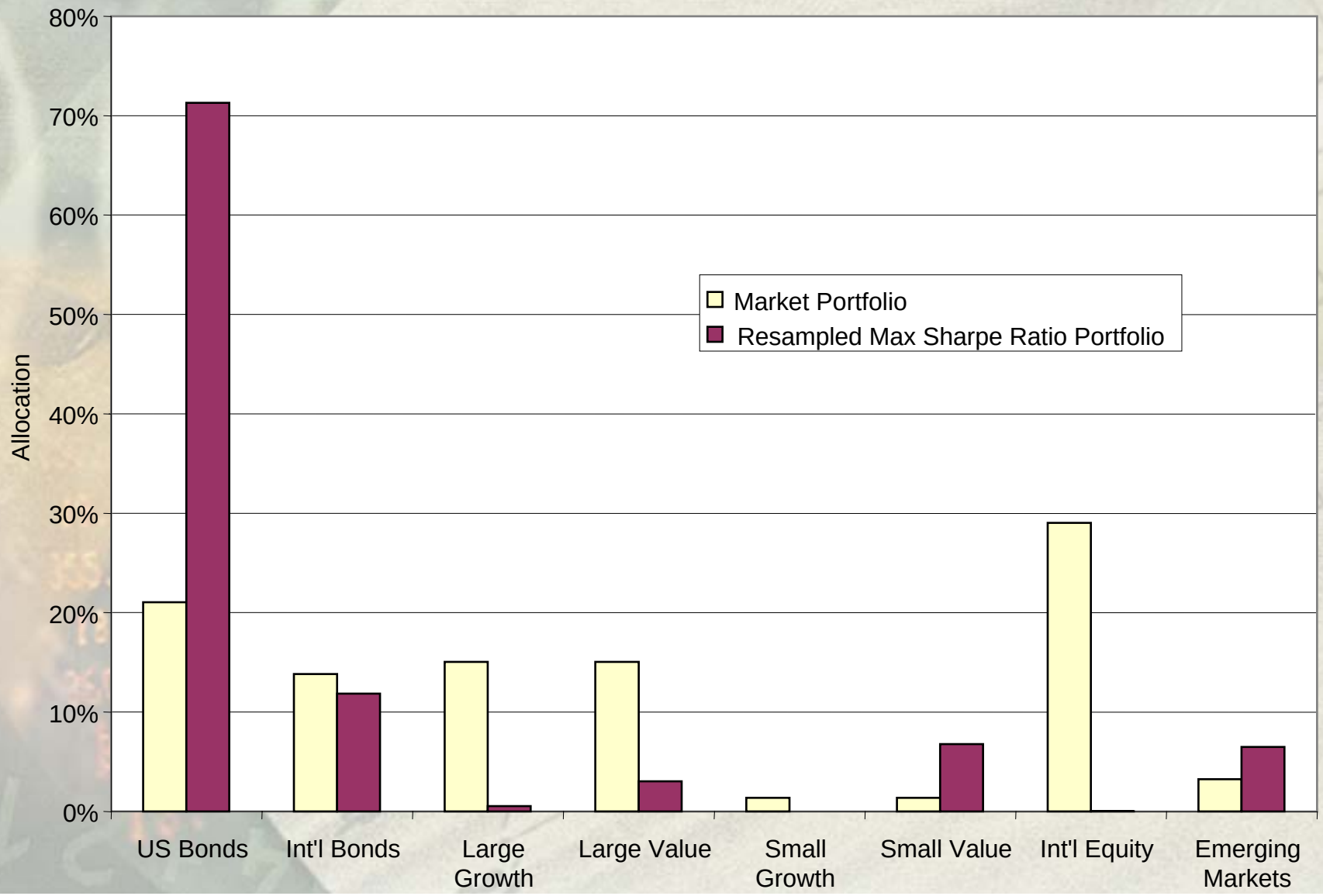




The Market Portfolio: A Neutral Portfolio

	Weight
US Bonds	21%
Int'l Bonds	14%
Large Growth	15%
Large Value	15%
Small Growth	1%
Small Value	1%
Int'l Equity	29%
Emerging Markets	3%

Using Resampling Means Taking an Unintentional Active Risk



Resampling results in taking active risk—why take bets without a reason?

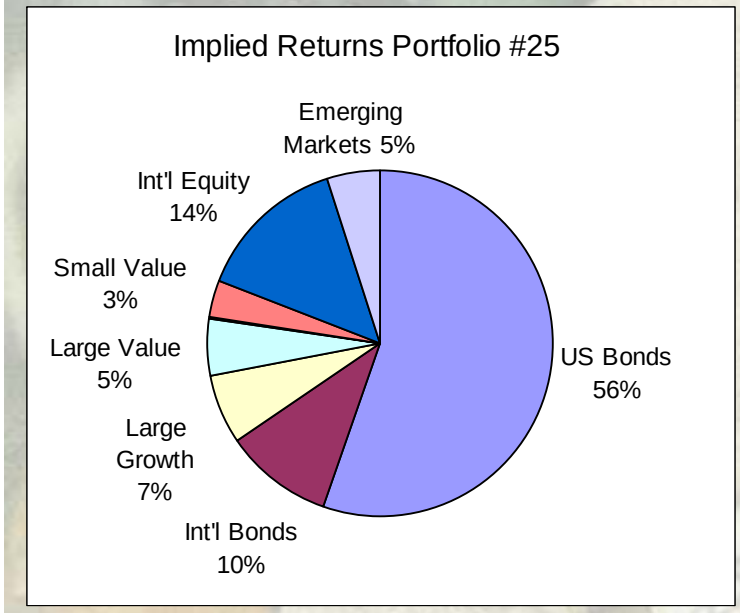
The Black-Litterman Model: A Better Way to Take Active Risk

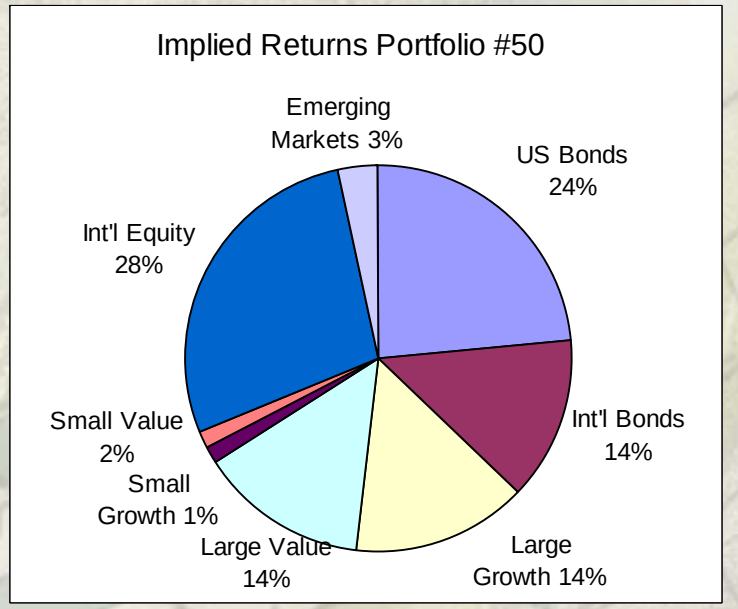
- Black-Litterman starts with the Implied Returns, which come from the market portfolio and are a neutral starting point.
- If you want to take a bet away from the market portfolio, Black-Litterman allows you to incorporate Views.
- The Black-Litterman mixed estimation technique incorporates views so that the active risk you take makes sense and reflects your views.

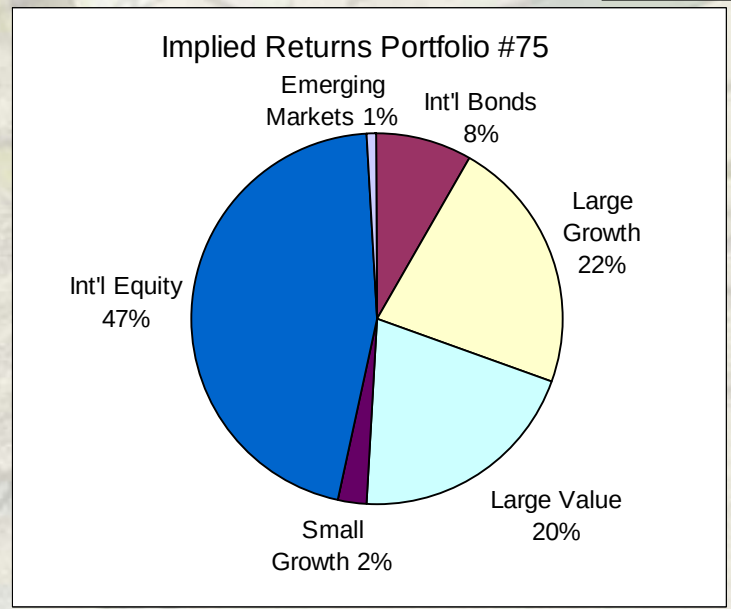
Implied Returns as Forecasts

- The Implied Returns make excellent forecasts for MVO in the absence of views.
- Using the Implied Returns with MVO results in intuitive portfolios.

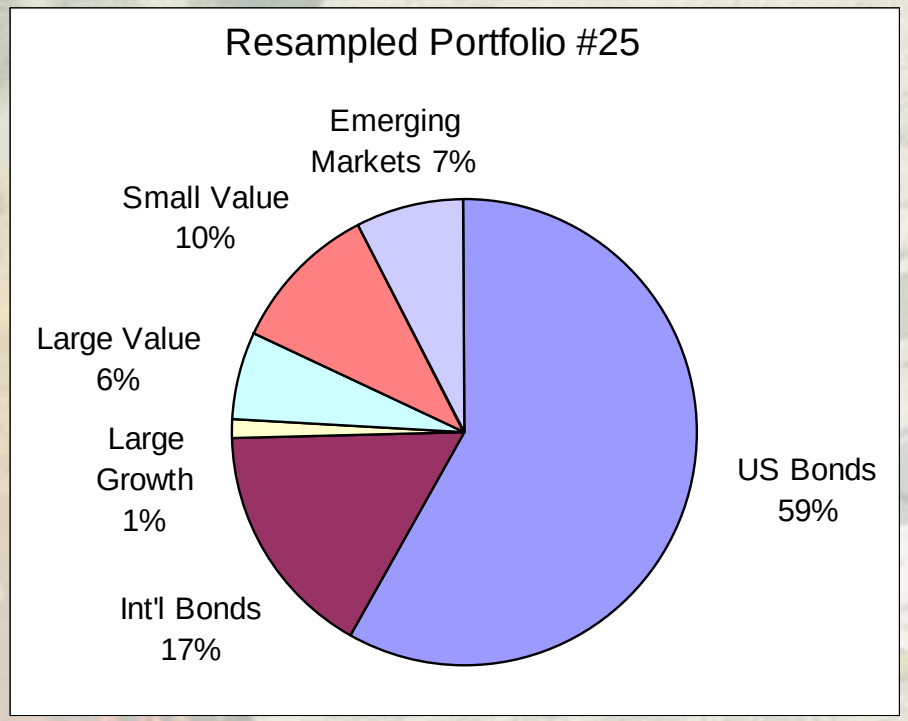
Portfolios Created Using the Implied Returns Make Sense

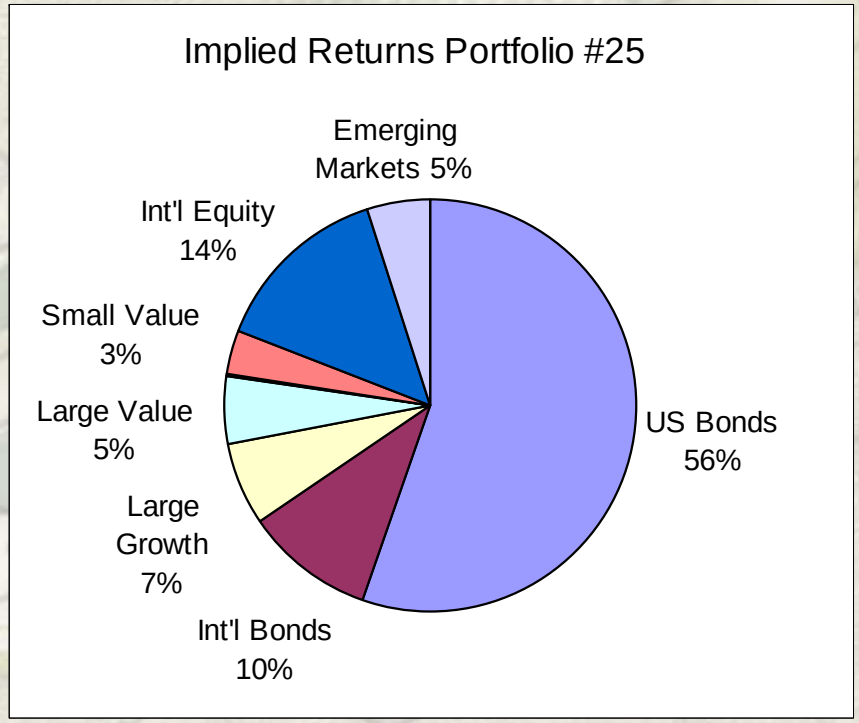




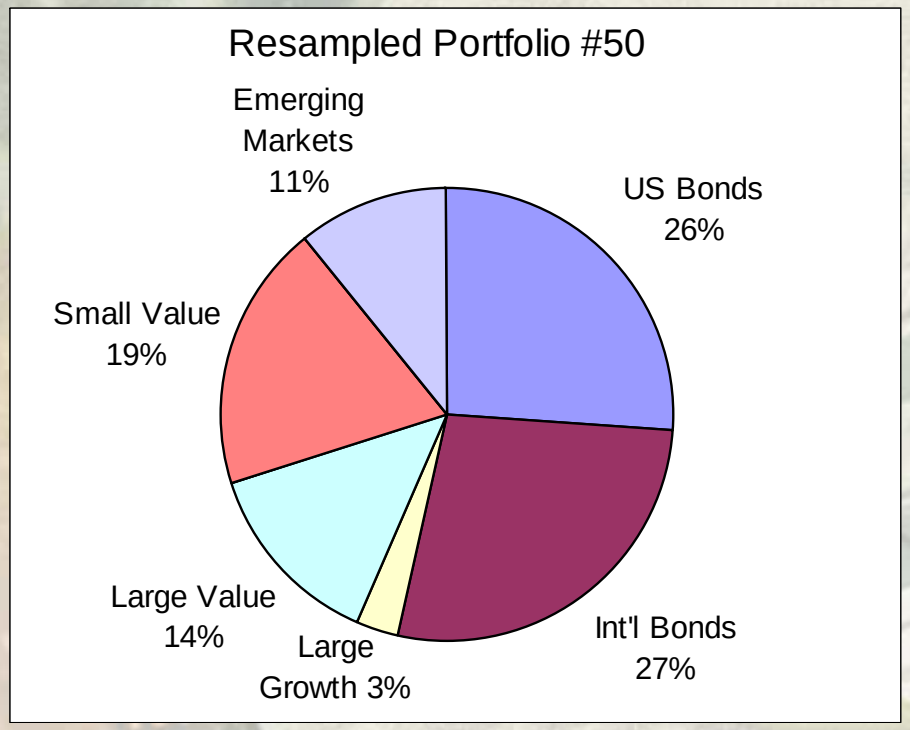


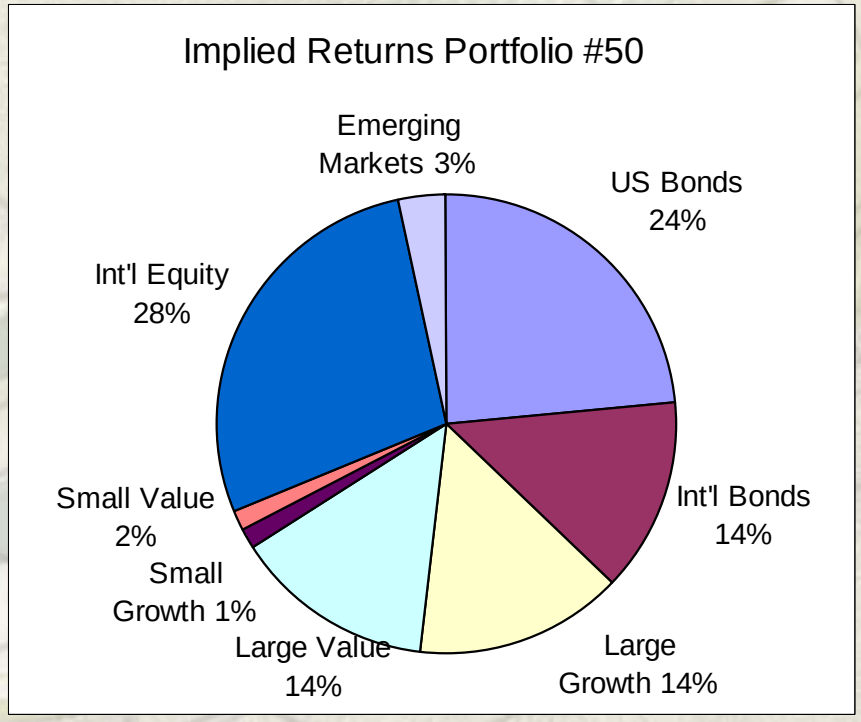
Portfolio # 25



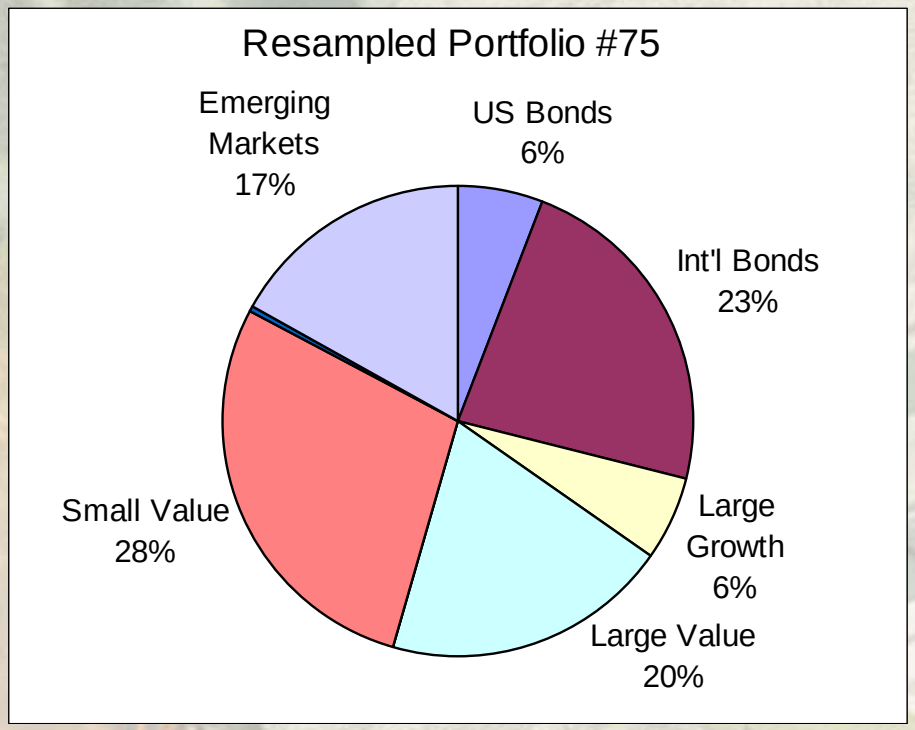


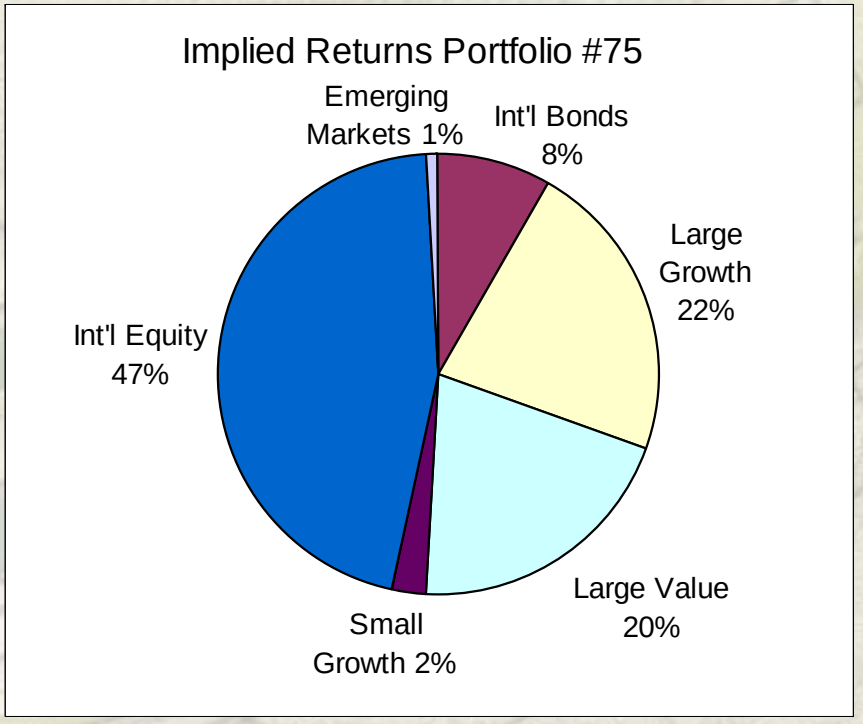
Portfolio #50



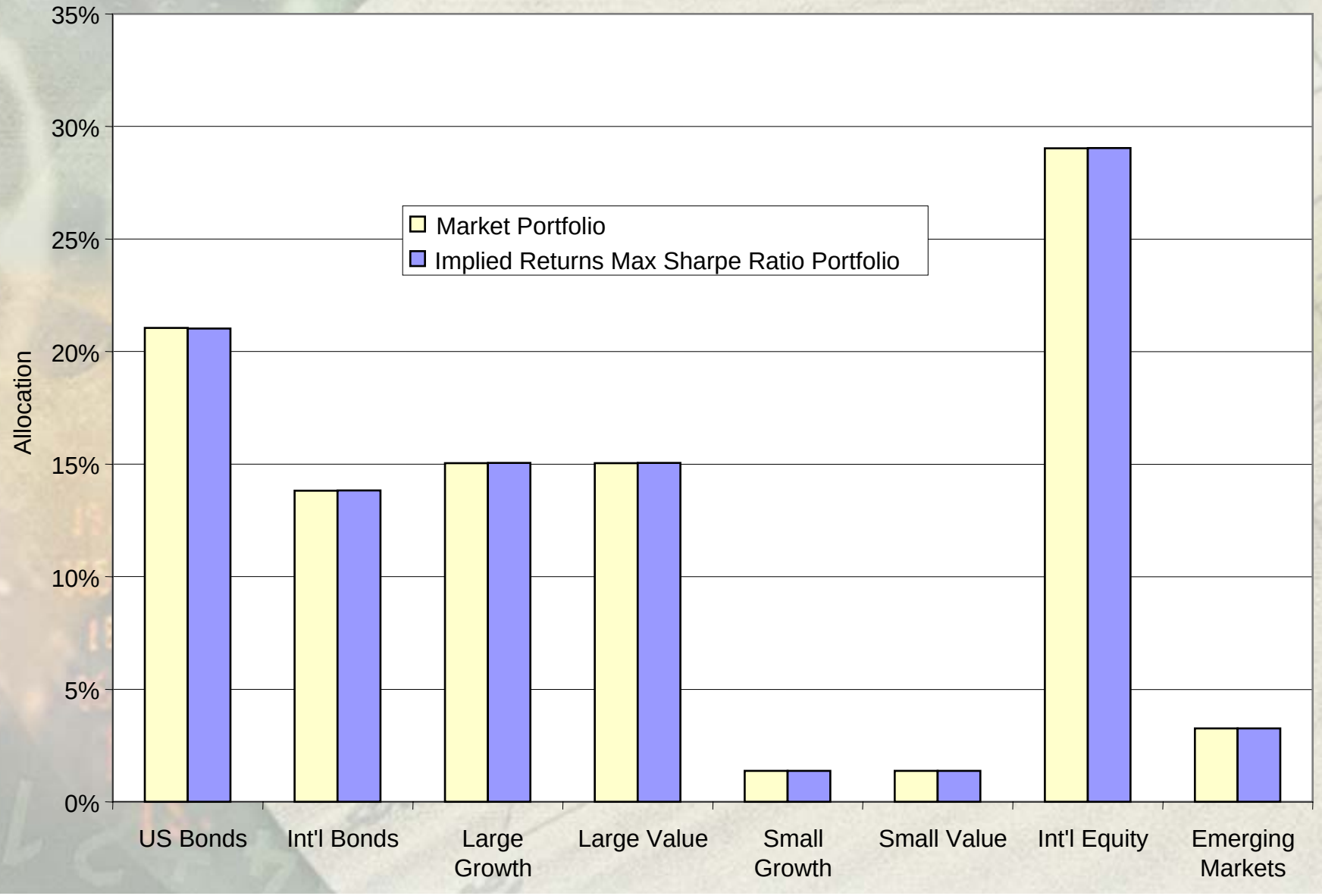


Portfolio #75





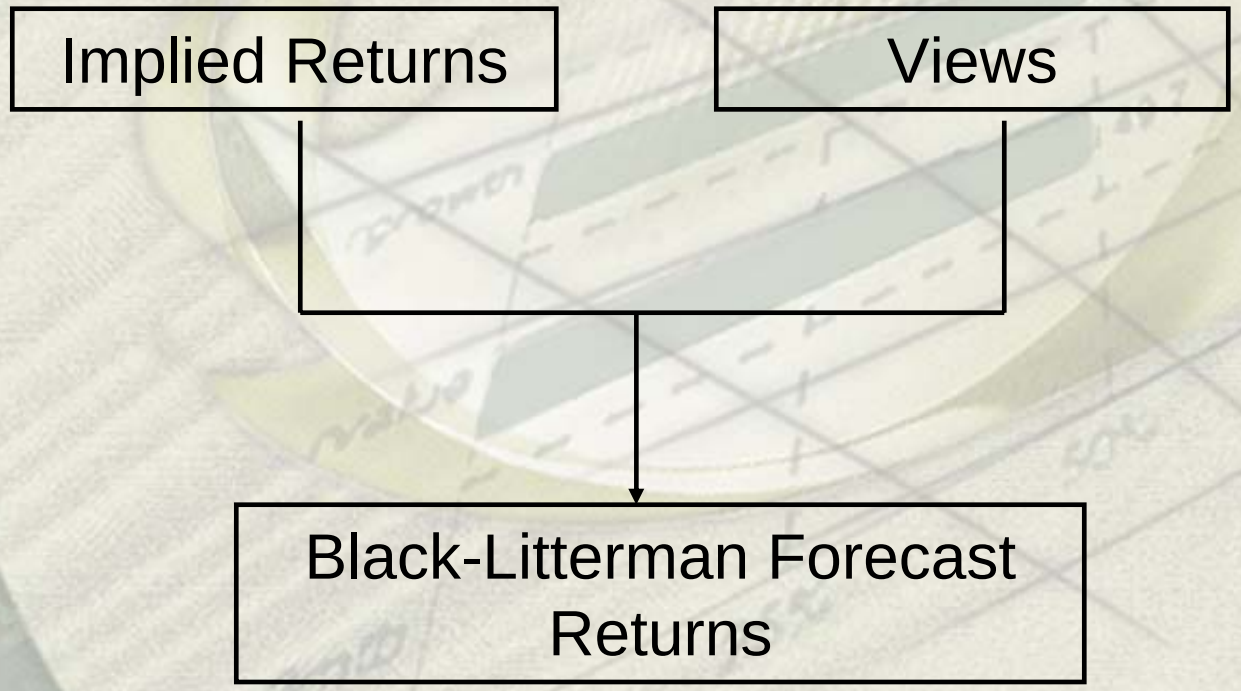
The Implied Returns are a Neutral Starting Point

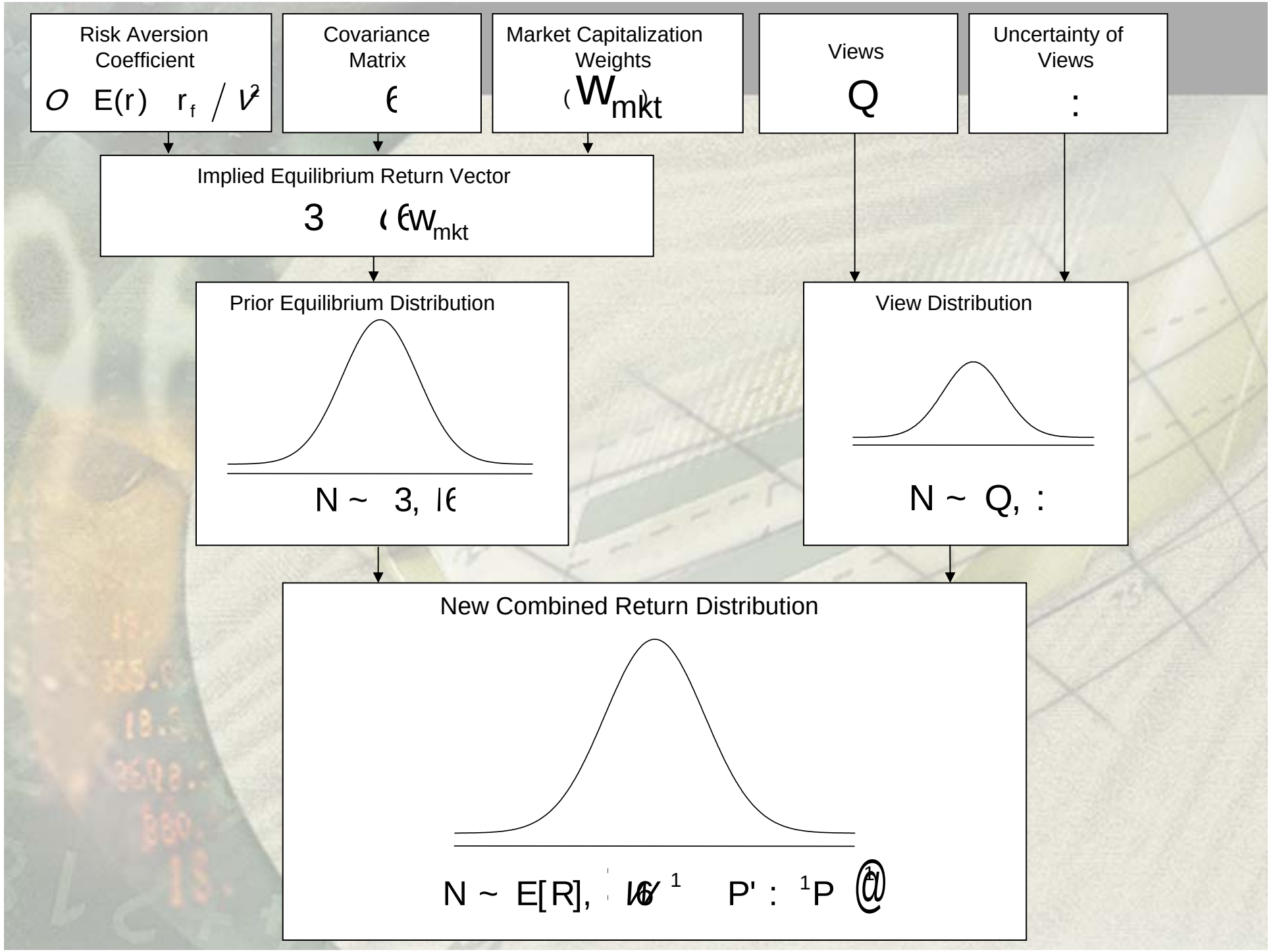


Views Allow You to Take Intentional Active Risk

- Views allow you to take an active risk away from the market portfolio.
- Views only have to be expressed for those assets about which you have special knowledge or strong opinions.

The Implied Returns are Combined with Your Views to Create New Black-Litterman Forecasts

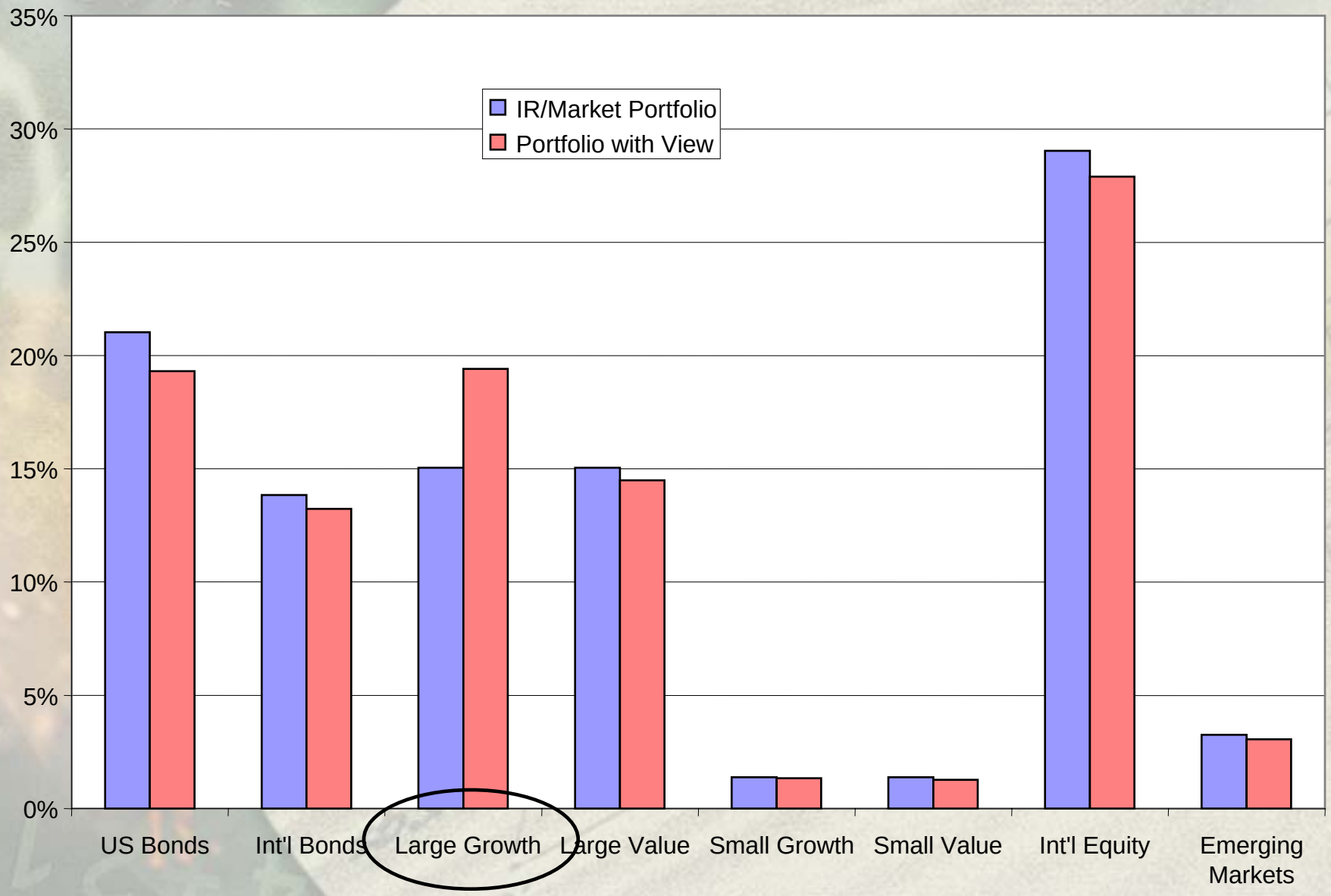



Risk Aversion Coefficient
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Covariance Matrix
(
Market Capitalization Weights
(W_mkt)
Views
Q
Uncertainty of Views
:
Implied Equilibrium Return Vector
3 ((w_mkt
Prior Equilibrium Distribution
N ~ 3, (
View Distribution
N ~ Q, :
New Combined Return Distribution
N ~ E[R], [(( )-1 P' : -1P (

Sample View

- Sample View: Large Growth will have an annualized return of 14% (Implied Return is 12.2%).

An Active Bet Toward Large Growth



Conclusion

- Both Black-Litterman and Resampling result in diversified portfolios.
- Black-Litterman also provides intuitive portfolios.
- Black-Litterman allows you to take purposeful active risk with the use of Views.

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